# **Methods Revision Notes –** Chapter 1

Differentiation Basics

$y=ax^{2}+bx^{1}+c$, $y^{'}=2ax^{1}+bx^{0}+$0 etc.

For $y=5x^{4},$find $eq^{n}$of the tangent line at the point (2,80)

$$y^{'}=20x^{3}, y^{'}\left(2\right)=20\left(2\right)^{3}=160$$

$y=160x+c$, substitute (2,80) ⟹$80=160\left(2\right)+c, c=\frac{1}{4}$

$$y=160x+\frac{1}{4}$$

Product Rule

$$y=f(x)×g(x)$$

$$y^{'}=f\left(x\right) g^{'}\left(x\right)+g\left(x\right) f'(x)$$

Quotient Rule

$$y=\frac{f(x)}{g(x)}$$

$$y^{'}=\frac{g\left(x\right) f^{'}\left(x\right)-f\left(x\right) g'(x)}{[g(x)]^{2}}$$

Chain Rule

If $y=f(u)$ and $u=g(x)$ then $\frac{dy}{dx}=\frac{dy}{du}×\frac{du}{dx}$

If $y=3u^{2}-1$ and $u=2x+3$

$$\frac{dy}{dx}=\frac{dy}{du}×\frac{du}{dx}$$

$$=6u×2$$

$$=12u$$

Substitute $=12\left(2x+3\right)$

 $=24x+36$ $ $

$$y=\left(4x^{9}-3\right)^{27}$$

$$y^{'}=27(36x^{8})\left(4x^{9}-3\right)$$

 $=972x^{8}\left(4x^{9}-3\right)^{26}$

Determining the nature of stationary points

For a function $f(x)$, if $f'(a)$, then there is a stationary point at $x=a$

Test 1: The Sign Test

A maximum turning point: A minimum turning point:

|  |  |  |  |
| --- | --- | --- | --- |
| $$x$$ | $$a^{-}$$ | $$a$$ | $$a^{+}$$ |
| $$f'(x)$$ | $$+$$ | $$0$$ | $$-$$ |

|  |  |  |  |
| --- | --- | --- | --- |
| $$x$$ | $$a^{-}$$ | $$a$$ | $$a^{+}$$ |
| $$f'(x)$$ | $$-$$ | $$0$$ | $$+$$ |

A horizontal point of inflection:

|  |  |  |  |
| --- | --- | --- | --- |
| $$x$$ | $$a^{-}$$ | $$a$$ | $$a^{+}$$ |
| $$f'(x)$$ | $$-$$ | $$0$$ | $$-$$ |

|  |  |  |  |
| --- | --- | --- | --- |
| $$x$$ | $$a^{-}$$ | $$a$$ | $$a^{+}$$ |
| $$f'(x)$$ | $$+$$ | $$0$$ | $$+$$ |

Test 2: The Second Derivative Test

If $f^{'}\left(a\right)=0 $and $f^{''}\left(a\right)>0$, the curve is concave up and $(a, f\left(x\right))$ is a minimum point

If $f^{'}\left(a\right)=0 $and $f^{''}\left(a\right)<0$, the curve is concave down and $(a, f\left(x\right))$ is a maximum point

If $f^{''}\left(a\right)=0$, then check for a point of inflection, i.e. if $f''(a^{-})$ and $f''(a^{+})$ have opposite signs. If $f'(a)$ also $=0$, then $f(a$) is a horizontal point of inflection

Where C(x) gives the cost per item, and x is the number of items, the average cost for C(a) items is $\frac{C(a)}{x}$

Small Changes

Find the small change in V when r changes from 5 to 5.1 in the equation $V=\frac{4}{3}πr^{3}$

$$δV≈\frac{dV}{dr}×δr$$

$$δV≈\frac{dV}{dr}×δr, \frac{dV}{dr}=4πr^{2}, δr=0.1$$

$$ =4πr^{2}×0.1$$

$ =0.4πr^{2}$ $(substitute inital value for r, ie. r=5)$

$$ =0.4π(5)^{2}$$

$$ =10π$$

Find the percentage change in V when r changes by 5% in the equation $V=\frac{4}{3}πr^{3}$

$$Requires: \frac{δV}{V}$$

$$\frac{δr}{r}=5\%=0.05, δr=0.05r$$

$$δV≈\frac{dV}{dr}×δr, \frac{dV}{dr}=4πr^{2}$$

$$ =4πr^{2}×0.05r$$

$$ =0.2πr^{3}$$

$$\frac{δV}{V}=\frac{0.2πr^{3}}{\frac{4}{3}πr^{3}}$$

$ =0.15$ ⟹ 15%